Kinematics & Dynamics of Linkages

Lecture 16: Acceleration and Jerk

Acceleration Analysis

- Acceleration
  - Rate of change of velocity with respect to time
  - May be Angular or Linear

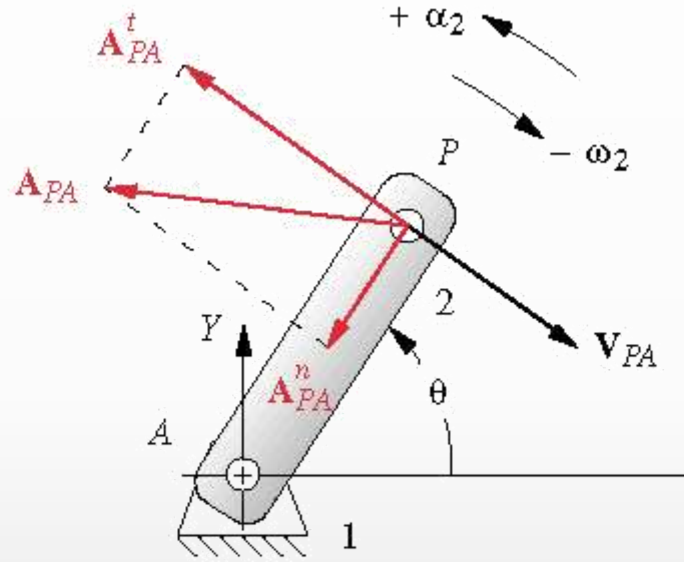
$$\text{Angular Acceleration} = \alpha = \frac{d\omega}{dt}$$

$$\text{Linear Acceleration} = A = \frac{dV}{dt}$$

Recap

- Position of point P (with respect to A)

$$R_{PA} = pe^{j\theta}$$



- Velocity of point P (with respect to A)

$$V_{PA} = \frac{dR_{PA}}{dt} = pje^{j\theta}\frac{d\theta}{dt} = p\omega je^{j\theta} = p\omega e^{j(\theta+90^0)}$$

- ω is + ccw and – cw

Acceleration

$$A_{PA} = \frac{dV_{PA}}{dt} = \frac{d\left(p\omega je^{j\theta}\right)}{dt}$$

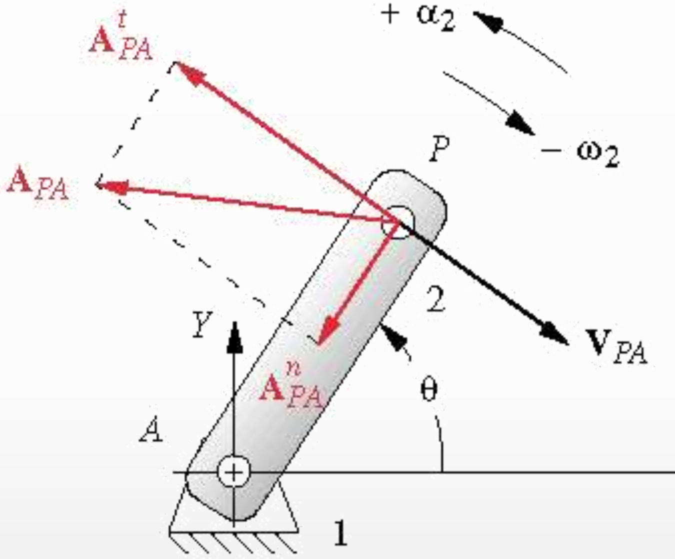
$$A_{PA} = jp\left(e^{j\theta}\frac{d\omega}{dt} + \omega je^{j\theta}\frac{d\theta}{dt}\right)$$

$$A_{PA} = p\alpha je^{j\theta} - p\omega^2 e^{j\theta}$$

$$A_{PA} = A_{PA}^t + A_{PA}^n$$

A_{PA}^t the tangential component

A_{PA}^n the normal component



Acceleration – Relative

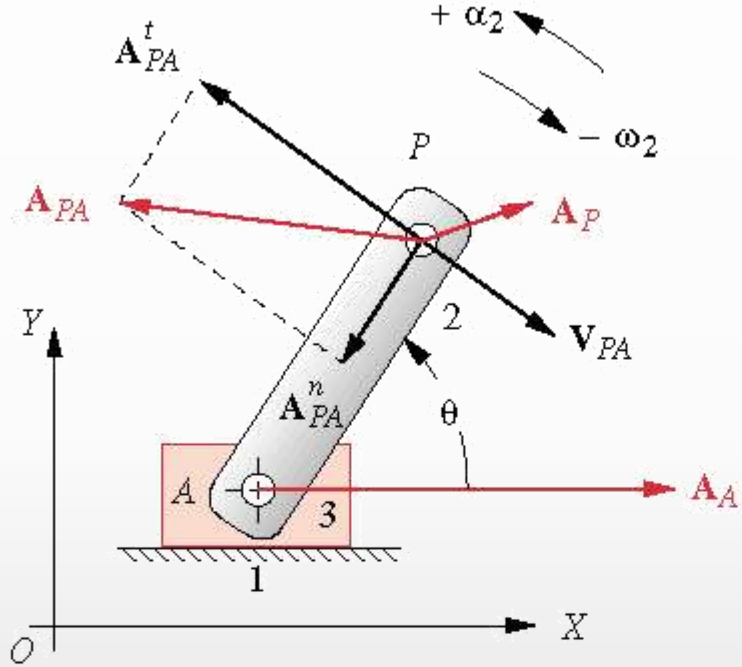
$$A_P = A_A + A_{PA}$$

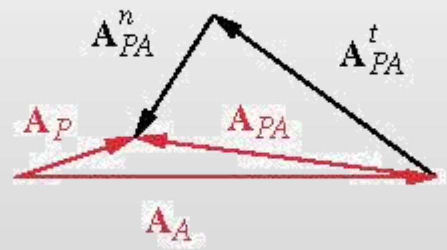
$$A_{PA} = p\alpha j e^{j\theta} - p\omega^2 e^{j\theta}$$

$$A_{PA} = A_{PA}^t + A_{PA}^n$$

$$A_{PA}^t = p\alpha j e^{j\theta}$$

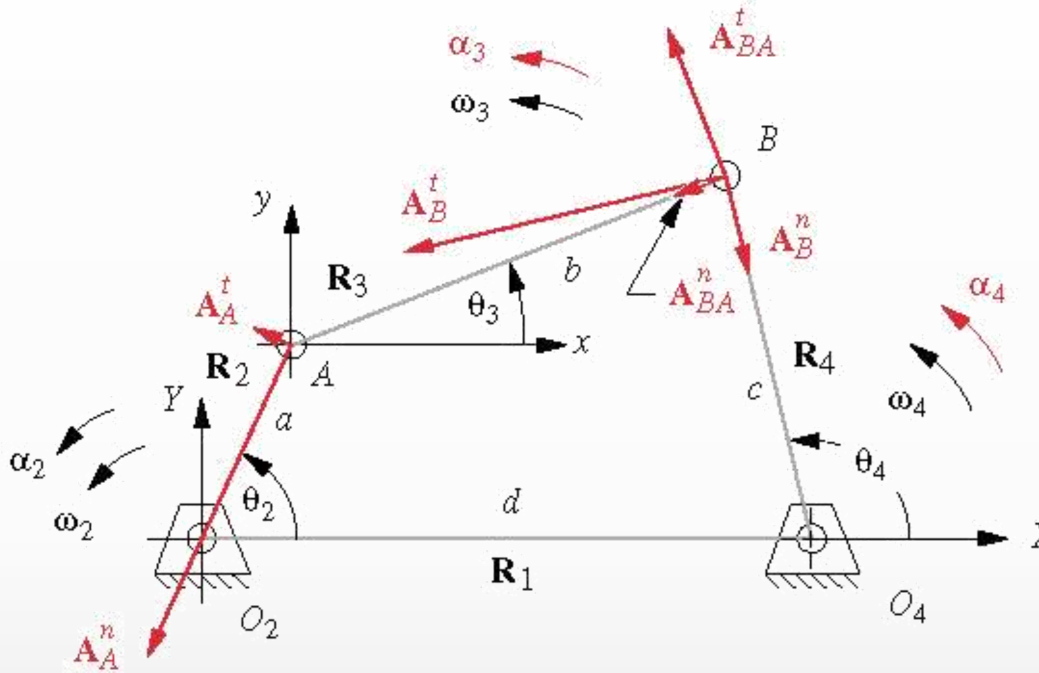
$$A_{PA}^n = -p\omega^2 e^{j\theta}$$





4Bar Linkage

- Find α_3 and α_4 given α_2



$$R_2 + R_3 - R_4 - R_1 = 0$$

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0 \quad \theta_1 = 0$$

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - d = 0$$

4Bar Linkage

Derive the equation twice and rearrange to get

$$\underbrace{\left(\overbrace{a\alpha_2 je^{j\theta_2}}^{A_{A,tangential}} - \overbrace{a\omega_2^2 e^{j\theta_2}}^{A_{A,normal}}\right)}_{A_A} + \underbrace{\left(\overbrace{b\alpha_3 je^{j\theta_3}}^{A_{BA,tangential}} - \overbrace{b\omega_3^2 e^{j\theta_3}}^{A_{BA,normal}}\right)}_{A_{BA}} - \underbrace{\left(\overbrace{c\alpha_4 je^{j\theta_4}}^{A_{B,tangential}} - \overbrace{c\omega_4^2 e^{j\theta_4}}^{A_{B,normal}}\right)}_{A_B} = 0$$

$$A_A + A_{BA} - A_B = 0$$

Two equations (real & Imaginary) and unkowns: α_3 and α_4

4Bar Linkage

Solution: $\alpha_3 = \dfrac{CD - AF}{AE - BD}$ $\alpha_4 = \dfrac{CE - BF}{AE - BD}$

Where

$$A = c\sin\theta_4$$

$$B = b\sin\theta_3$$

$$C = a\alpha_2\sin\theta_2 + a\omega_2^2\cos\theta_2 + b\omega_3^2\cos\theta_3 - c\omega_4^2\cos\theta_4$$

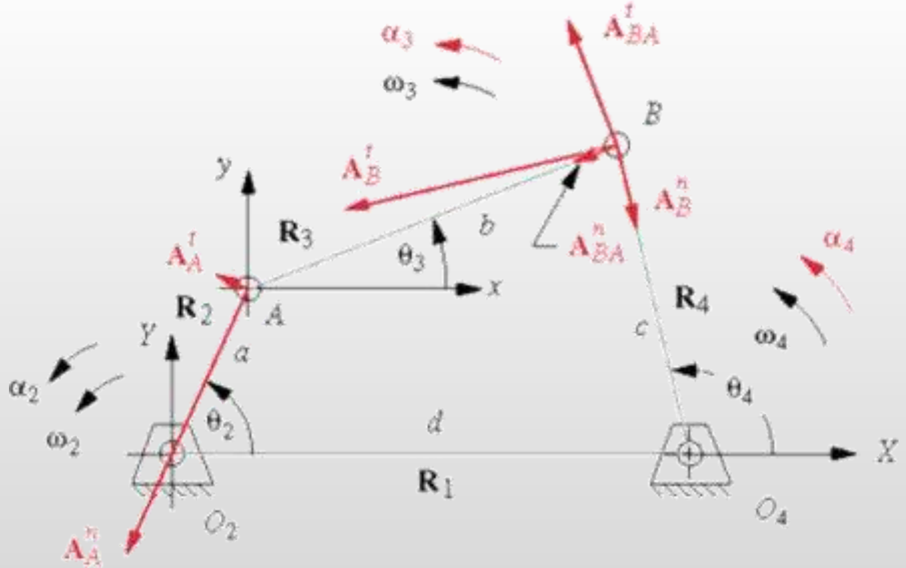
$$D = c\cos\theta_4$$

$$E = b\cos\theta_3$$

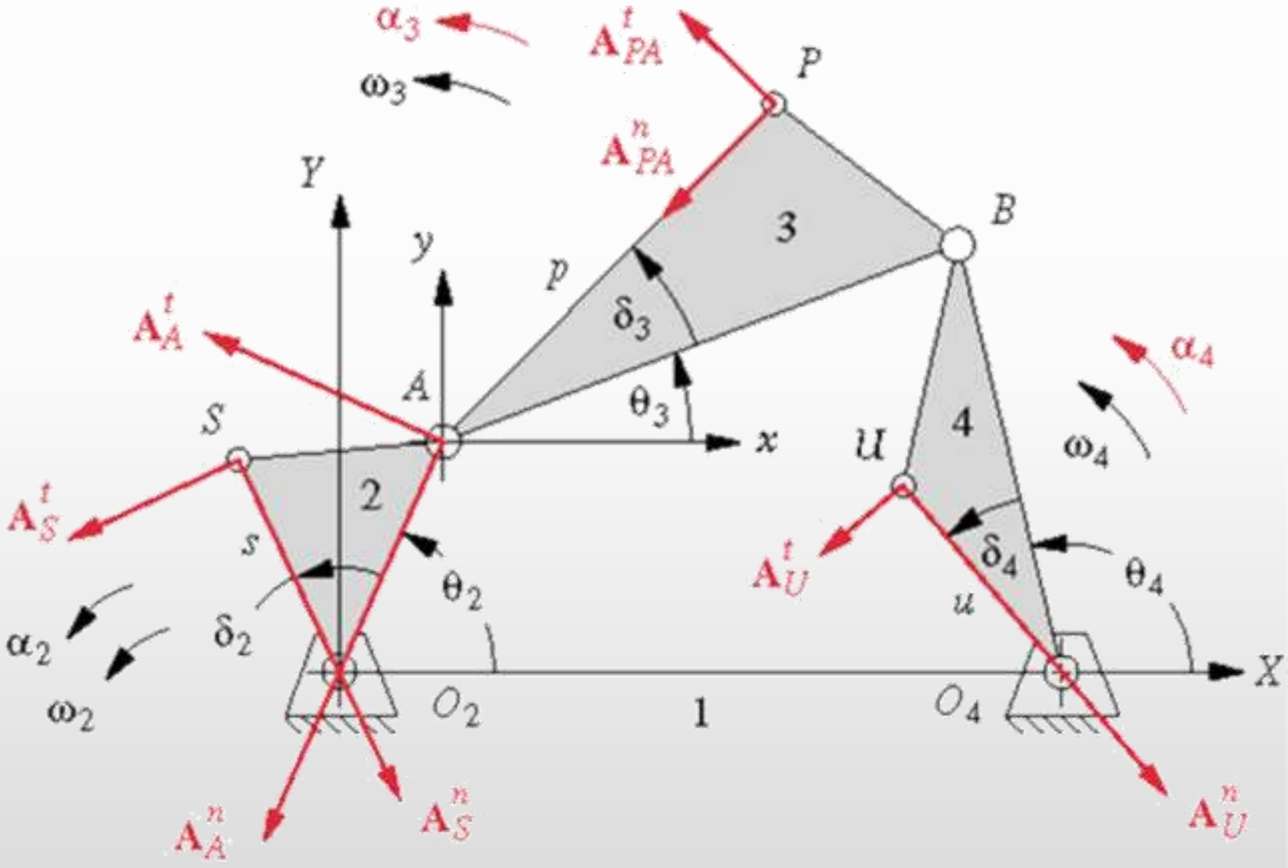
$$F = a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 - b\omega_3^2\sin\theta_3 + c\omega_4^2\sin\theta_4$$

4Bar Linkage Steps

- **Step 1:** Solve for position > $\theta_3 \quad and \quad \theta_4$
- **Step 2:** Solve for velocity > $\omega_3 \quad and \quad \omega_4$
- **Step 3:** Solve for acceleration > $\alpha_3 \quad and \quad \alpha_4$



4Bar Linkage – Other Points



4Bar Linkage – Point (S)

- Position $R_S = se^{j(\theta_2+\delta_2)} = s\left(\cos\left(\theta_2+\delta_2\right)+j\sin\left(\theta_2+\delta_2\right)\right)$

- Velocity $V_S = \frac{dR_S}{dt} = jse^{j(\theta_2+\delta_2)}\omega_2 = s\omega_2\left(-\sin(\theta_2+\delta_2)+j\cos(\theta_2+\delta_2)\right)$

- Acceleration $A_S = \frac{dV_S}{dt}$

$$A_S = s\alpha_2\left(-\sin(\theta_2+\delta_2)+j\cos(\theta_2+\delta_2)\right)-s\omega_2^2\left(\cos(\theta_2+\delta_2)+j\sin(\theta_2+\delta_2)\right)$$

4Bar Linkage – Point (U)

- Position $R_U = ue^{j(\theta_4+\delta_4)} = u\left(\cos\left(\theta_4+\delta_4\right)+j\sin\left(\theta_4+\delta_4\right)\right)$
- Velocity $V_U = \frac{dR_U}{dt} = jue^{j(\theta_4+\delta_4)}\omega_4 = u\omega_4\left(-\sin(\theta_4+\delta_4)+j\cos(\theta_4+\delta_4)\right)$
- Acceleration $A_U = \frac{dV_U}{dt}$

$$A_U = u\alpha_4\left(-\sin(\theta_4+\delta_4)+j\cos(\theta_4+\delta_4)\right)-u\omega_4^2\left(\cos(\theta_4+\delta_4)+j\sin(\theta_4+\delta_4)\right)$$

4Bar Linkage – Point (P)

- Position $R_P = R_A + R_{PA}$

$$R_{PA} = p(\cos(\theta_3 + \delta_3) + j\sin(\theta_3 + \delta_3))$$

- Velocity $V_P = V_A + V_{PA}$

$$V_{PA} = jpe^{j(\theta_3 + \delta_3)}\omega_3 = p\omega_3\left(-\sin(\theta_3 + \delta_3) + j\cos(\theta_3 + \delta_3)\right)$$

- Acceleration $A_P = A_A + A_{PA}$

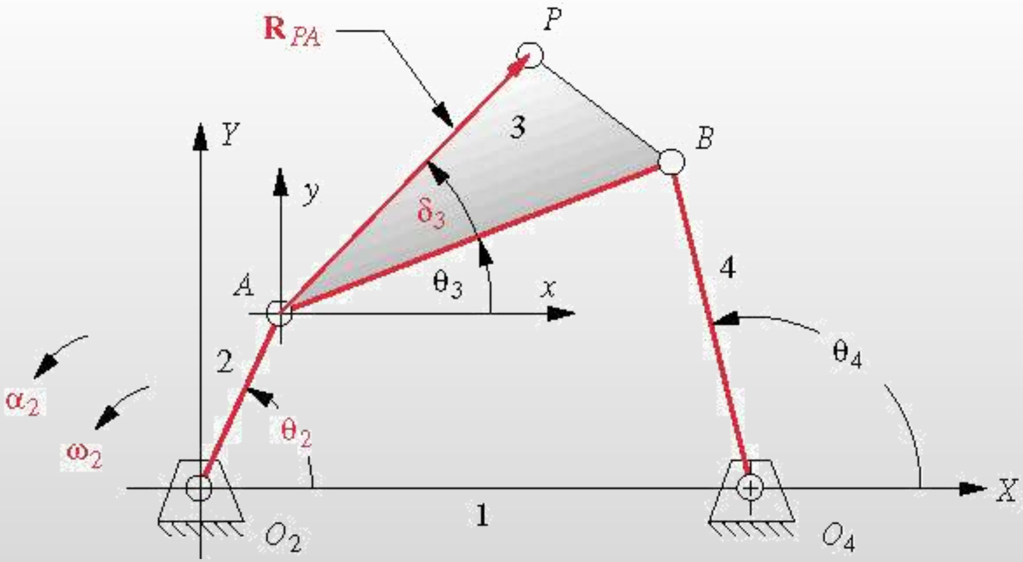
$$A_{PA} = p\alpha_3\left(-\sin(\theta_3 + \delta_3) + j\cos(\theta_3 + \delta_3)\right) - p\omega_3^2\left(\cos(\theta_3 + \delta_3) + j\sin(\theta_3 + \delta_3)\right)$$

4Bar Linkage – Example

- Link 1 = 8” = d
- Link 2 = 5” = a
- $\theta_2 = 75°$
- $\omega_2 = -50$ rad/s
- $\alpha_2 = 10$ rad/s^2

Link 3 = 8” = b
Link 4 = 6” = c
$R_{PA} = 9”$
$\delta_3 = 300°$

- Calculate $\mathbf{A_A}$, $\mathbf{A_B}$ and $\mathbf{A_P}$



4Bar Linkage – Example

- From Steps 1 and 2: $\theta_3 = 7.5^o \quad \theta_4 = 78.2^o \quad \omega_3 = 1.85\, rad/s \quad \omega_4 = -40.8\, rad/s$
- **Step 3**

$$A = c\sin\theta_4 = 5.873$$
$$B = b\sin\theta_3 = 1.044$$
$$C = a\alpha_2 \sin\theta_2 + a\omega_2^2 \cos\theta_2 + b\omega_3^2 \cos\theta_3 - c\omega_4^2 \cos\theta_4 = 1265.2$$
$$D = c\cos\theta_4 = 1.227$$
$$E = b\cos\theta_3 = 7.932$$
$$F = a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 - b\omega_3^2 \sin\theta_3 + c\omega_4^2 \sin\theta_4 = -2287.9$$

$$\alpha_3 = \frac{CD - AF}{AE - BD} = 331\, rad/s^2 \qquad \alpha_4 = \frac{CE - BF}{AE - BD} = 274\, rad/s^2$$

4Bar Linkage – Example

- Find A_A, A_B and A_P

$$A_A = A_S \quad \left(s = a \quad and \quad \delta_2 = 0\right)$$

$$A_S = s\alpha_2\left(-\sin(\theta_2+\delta_2)+j\cos(\theta_2+\delta_2)\right)-s\omega_2^2\left(\cos(\theta_2+\delta_2)+j\sin(\theta_2+\delta_2)\right)$$

$$A_A = a\alpha_2\left(-\sin\theta_2 + j\cos\theta_2\right)-a\omega_2^2\left(\cos\theta_2 + j\sin\theta_2\right)$$

$$A_A = -3283.6 - j\,12061.1 \quad in/s^2$$

$$A_B = A_U \quad \left(u = c \quad and \quad \delta_4 = 0\right)$$

$$A_U = u\alpha_4\left(-\sin(\theta_4+\delta_4)+j\cos(\theta_4+\delta_4)\right)-u\omega_4^2\left(\cos(\theta_4+\delta_4)+j\sin(\theta_4+\delta_4)\right)$$

$$A_B = c\alpha_4\left(-\sin\theta_4 + j\cos\theta_4\right)-c\omega_4^2\left(\cos\theta_4 + j\sin\theta_4\right)$$

$$A_B = -3651.73 - j\,9440.57 \quad in/s^2$$

4Bar Linkage – Example

- Find A_A, A_B and A_P

$$A_P = A_A + A_{PA}$$

$$A_{PA} = p\alpha_3\left(-\sin(\theta_3+\delta_3)+j\cos(\theta_3+\delta_3)\right)-p\omega_3^2\left(\cos(\theta_3+\delta_3)+j\sin(\theta_3+\delta_3)\right)$$

$$A_{PA} = 2343.6 + j\,1838.6 \quad in/s^2$$

$$A_P = -940 - j\,10222.5 \quad in/s^2$$

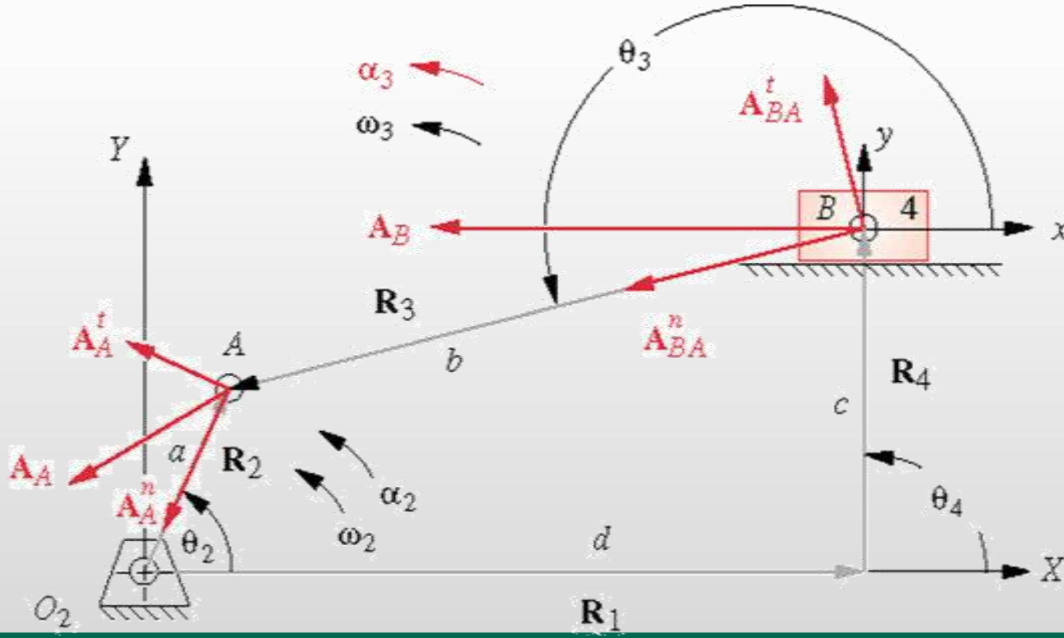
Slider Crank

- Find α_3 and $\ddot{d}$ given α_2 using the vector loop equation

$$R_2 - R_3 - R_4 - R_1 = 0$$

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0 \quad \theta_1 = 0^o \quad \theta_4 = 90^o$$

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j90^o} - d = 0$$



Slider Crank

- Derive twice and arrange

$$\underbrace{\left(\overbrace{a\alpha_2 je^{j\theta_2}}^{A_{A,tangential}} \overbrace{-a\omega_2^2 e^{j\theta_2}}^{A_{A,normal}}\right)}_{A_A} - \underbrace{\left(\overbrace{b\alpha_3 je^{j\theta_3}}^{A_{AB,tangential}} \overbrace{-b\omega_3^2 e^{j\theta_3}}^{A_{AB,normal}}\right)}_{A_{AB}} - \underbrace{\ddot{d}}_{A_B} = 0$$

$A_A - A_{AB} - A_B = 0$

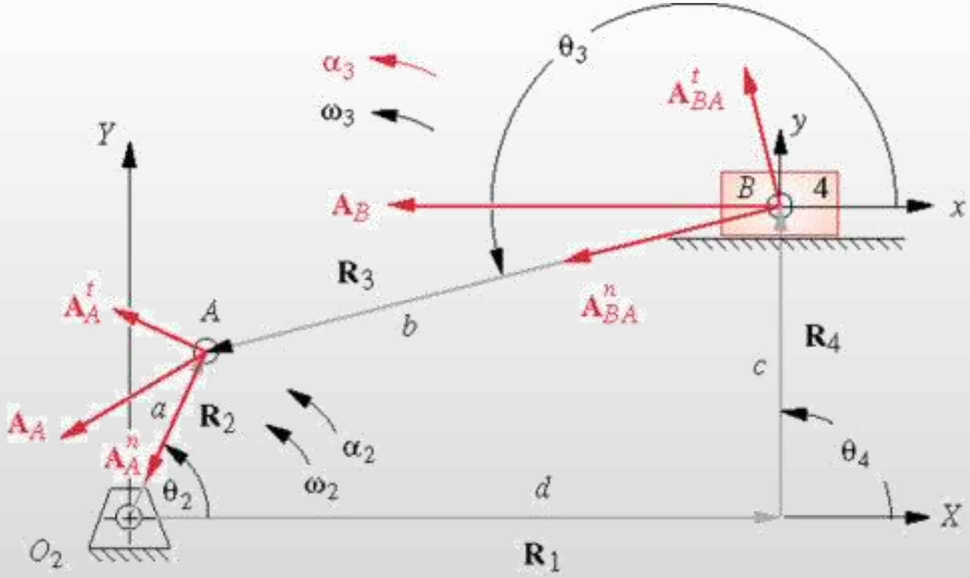
- Solution

$$\alpha_3 = \left(a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 + b\omega_3^2 \sin\theta_3\right) / b\cos\theta_3$$

$$\ddot{d} = -a\alpha_2 \sin\theta_2 - a\omega_2^2 \cos\theta_2 + b\alpha_3 \sin\theta_3 + b\omega_3^2 \cos\theta_3$$

Slider Crank Steps

- **Step 1:** Solve for position > $\theta_3 \quad and \quad d$
- **Step 2:** Solve for velocity > $\omega_3 \quad and \quad \dot{d}$
- **Step 3:** Solve for acceleration > $\alpha_3 \quad and \quad \ddot{d}$



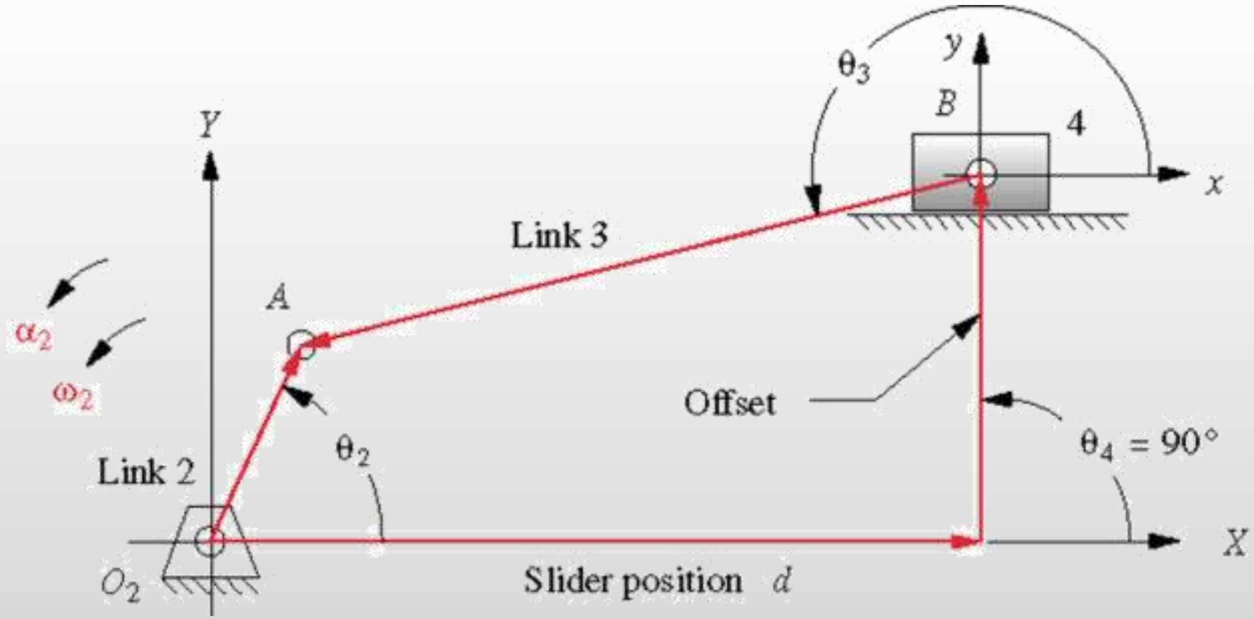
Slider Crank – Example

Link 2 = 3"
$\theta_2 = -30°$

Link 3 = 8"
$\omega_2 = -15$ rad/s

Offset = 2"
$\alpha_2 = -10$ rad/s^2

Determine Accelerations: A_A and A_B



Slider Crank – Example

- From Steps 1 and 2: $\theta_3 = 205.9^o \quad \omega_3 = 5.42\ rad/s$
- **Step 3**

$$\alpha_3 = \left(a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 + b\omega_3^2 \sin\theta_3\right) \Big/ b\cos\theta_3$$

$$\alpha_3 = -29.0 \quad rad/s^2$$

$$\ddot{d} = -a\alpha_2 \sin\theta_2 - a\omega_2^2 \cos\theta_2 + b\alpha_3 \sin\theta_3 + b\omega_3^2 \cos\theta_3$$

$$\ddot{d} = -709.4 \quad in/s^2$$

Slider Crank – Example

- Find A_A and A_B

$$A_A = A_S \quad (s = a \quad and \quad \delta_2 = 0)$$

$$A_S = s\alpha_2\left(-\sin(\theta_2+\delta_2)+j\cos(\theta_2+\delta_2)\right)-s\omega_2^2\left(\cos(\theta_2+\delta_2)+j\sin(\theta_2+\delta_2)\right)$$

$$A_A = a\alpha_2\left(-\sin\theta_2+j\cos\theta_2\right)-a\omega_2^2\left(\cos\theta_2+j\sin\theta_2\right)$$

$$A_A = -599.56 - j\,311.51 \quad in/s^2$$

$$A_A = 675.65 \quad @ \quad 207.4^o$$

$$A_B = \ddot{d}$$

$$A_B = -709.4 \quad in/s^2$$

$$A_B = 709.4 \quad @ \quad 180^o$$

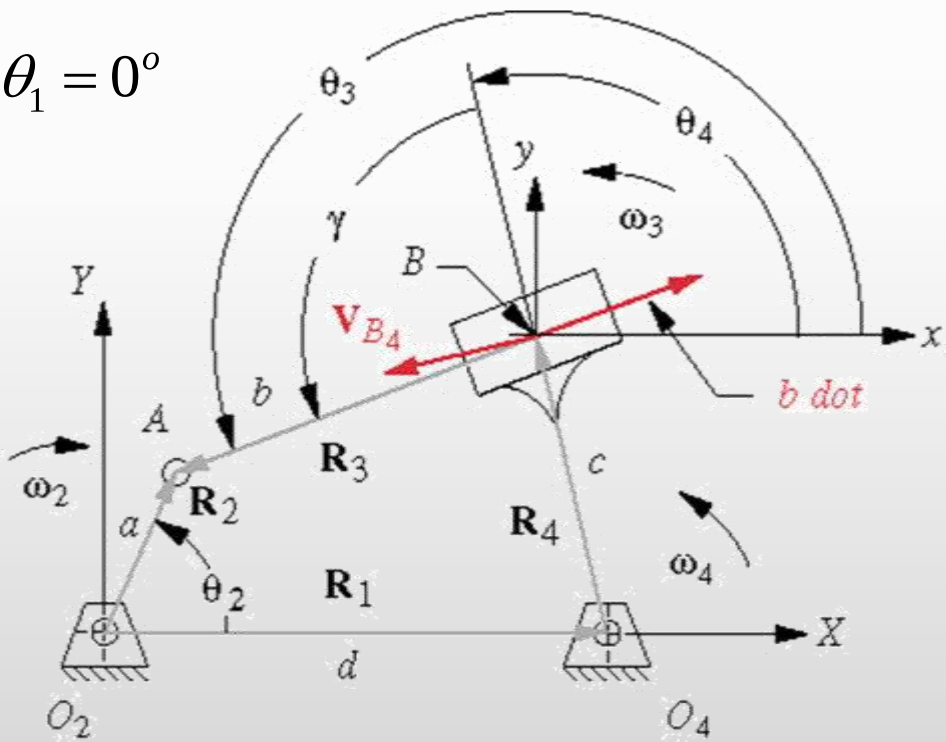
Inverted Slider Crank

- Find α_3, α_4 and $\ddot{b}$ given α_2 using the vector loop equation

$$R_2 - R_3 - R_4 - R_1 = 0$$

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0 \quad \theta_1 = 0^o$$

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - d = 0$$



Inverted Slider Crank

- Derive twice and arrange

$$\underbrace{\Big(\overbrace{ja\alpha_2 e^{j\theta_2}}^{A_{A,tangential}} - \overbrace{a\omega_2^2 e^{j\theta_2}}^{A_{A,normal}}\Big)}_{A_A} - \underbrace{\Big(\overbrace{b\alpha_3 je^{j\theta_3}}^{A_{AB,tangential}} - \overbrace{b\omega_3^2 e^{j\theta_3}}^{A_{AB,normal}} + \overbrace{2\dot{b}\omega_3 je^{j\theta_3}}^{A_{AB,coriolis}} + \overbrace{\ddot{b}e^{j\theta_3}}^{A_{AB,slip}}\Big)}_{A_{AB}} - \underbrace{\Big(\overbrace{c\alpha_4 je^{j\theta_4}}^{A_{B,tangential}} - \overbrace{c\omega_4^2 e^{j\theta_4}}^{A_{B,normal}}\Big)}_{A_B} = 0$$

A_A - A_{AB} - A_B = 0

- Solution: Angular acceleration and slip acceleration

$$\alpha_3 = \alpha_4 = \frac{a\left[\alpha_2 \cos(\theta_3 - \theta_2) + \omega_2^2 \sin(\theta_3 - \theta_2)\right] + c\omega_4^2 \sin(\theta_4 - \theta_3) - 2\dot{b}\omega_3}{b + c\cos(\theta_3 - \theta_4)}$$

$$\ddot{b} = \frac{\left\{ \begin{array}{c} a\omega_2^2\left[b\cos(\theta_3 - \theta_2) + c\cos(\theta_4 - \theta_2)\right] + a\alpha_2\left[b\sin(\theta_2 - \theta_3) - c\sin(\theta_4 - \theta_2)\right] \\ + 2\dot{b}c\omega_4 \sin(\theta_4 - \theta_3) - \omega_4^2\left[b^2 + c^2 + 2bc\cos(\theta_4 - \theta_3)\right] \end{array} \right\}}{b + c\cos(\theta_3 - \theta_4)}$$

Inverted Slider Crank

- The linear accelerations at points A, B and AB (A with respect to B)

$$A_A = a\alpha_2\left(-\sin\theta_2 + j\cos\theta_2\right) - a\omega_2^2\left(\cos\theta_2 + j\sin\theta_2\right)$$

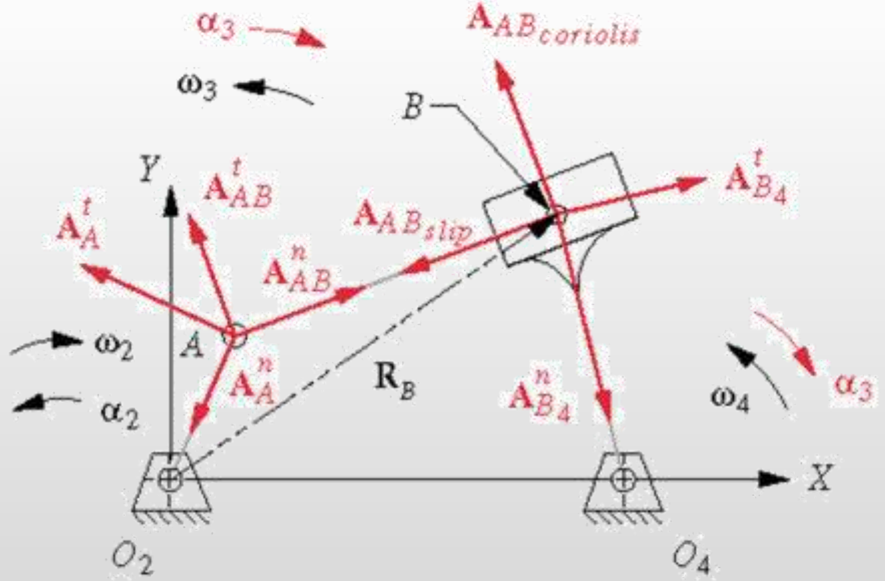
$$\begin{aligned} A_{BA} = -A_{AB} = b\alpha_3\left(\sin\theta_3 - j\cos\theta_3\right) + b\omega_3^2\left(\cos\theta_3 + j\sin\theta_3\right) \\ + 2\dot{b}\omega_3\left(\sin\theta_3 - j\cos\theta_3\right) - \ddot{b}\left(\cos\theta_3 + j\sin\theta_3\right) \end{aligned}$$

$$A_B = c\alpha_4\left(-\sin\theta_4 + j\cos\theta_4\right) - c\omega_4^2\left(\cos\theta_4 + j\sin\theta_4\right)$$

- Note: there is another method to find the linear accelerations independently from $\ddot{b}$ (Check book)

Inverted Slider Crank Steps

- **Step 1:** Solve for position > θ_3 and θ_4
- **Step 2:** Solve for velocity > ω_3, ω_4 and $\dot{b}$
- **Step 3:** Solve for acceleration > α_3, α_4 and $\ddot{b}$



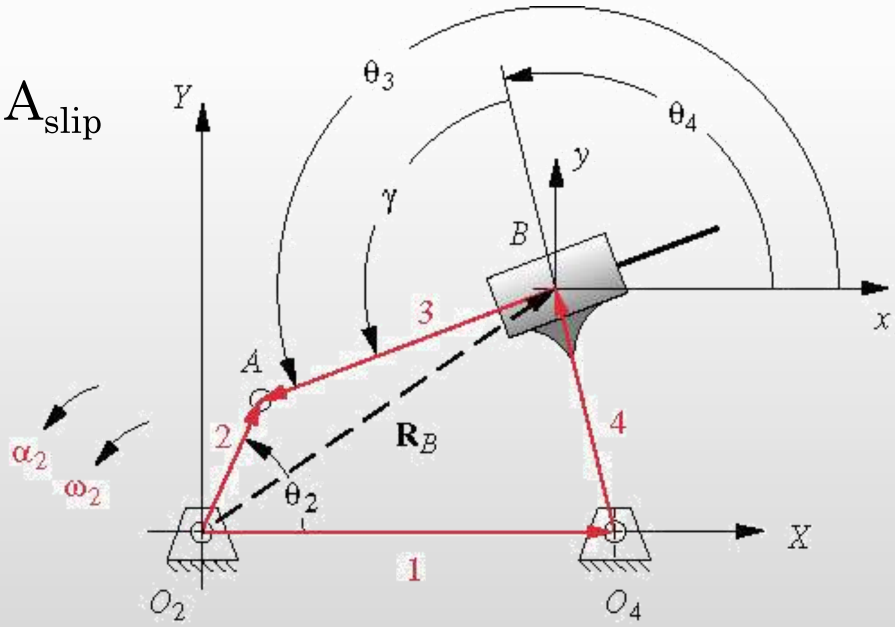
Inverted Slider Crank – Example

Link 1 = 3”
$\theta_2 = 45°$
$\gamma = 45^o$

Link 2 = 10”
$\omega_2 = 24$ rad/s

Link 4 = 6”
$\alpha_2 = 30$ rad/s^2

Determine Accelerations: A_A and A_{slip}



Inverted Slider Crank – Example

- From Steps 1 and 2:

$$\theta_3 = 91.4^o \quad \theta_4 = 46.4^o \quad b = 2.73'' \quad \omega_3 = \omega_4 = 23.7\,rad/s \quad \dot{b} = 83.77\,in/s$$

- **Step 3**

$$\alpha_3 = \alpha_4 = \frac{a\left[\alpha_2 \cos(\theta_3 - \theta_2) + \omega_2^2 \sin(\theta_3 - \theta_2)\right] + c\omega_4^2 \sin(\theta_4 - \theta_3) - 2\dot{b}\omega_3}{b + c\cos(\theta_3 - \theta_4)}$$

$$\alpha_3 = \alpha_4 = -212.9\,rad/s^2$$

$$\ddot{b} = \frac{\left\{\begin{matrix} a\omega_2^2\left[b\cos(\theta_3 - \theta_2) + c\cos(\theta_4 - \theta_2)\right] + a\alpha_2\left[b\sin(\theta_2 - \theta_3) - c\sin(\theta_4 - \theta_2)\right] \\ + 2\dot{b}c\omega_4 \sin(\theta_4 - \theta_3) - \omega_4^2\left[b^2 + c^2 + 2bc\cos(\theta_4 - \theta_3)\right] \end{matrix}\right\}}{b + c\cos(\theta_3 - \theta_4)}$$

$$\ddot{b} = 3327.9\,in/s^2$$

Inverted Slider Crank – Example

- Find A_A and A_{slip}

$$A_A = A_S \quad \left(s = a \quad and \quad \delta_2 = 0\right)$$

$$A_S = s\alpha_2\left(-\sin(\theta_2+\delta_2) + j\cos(\theta_2+\delta_2)\right) - s\omega_2^2\left(\cos(\theta_2+\delta_2) + j\sin(\theta_2+\delta_2)\right)$$

$$A_A = a\alpha_2\left(-\sin\theta_2 + j\cos\theta_2\right) - a\omega_2^2\left(\cos\theta_2 + j\sin\theta_2\right)$$

$$A_A = -4285.06 - j\,3860.8 \quad in/s^2$$

$$A_A = 5767.8 \quad @ \quad 222^o$$

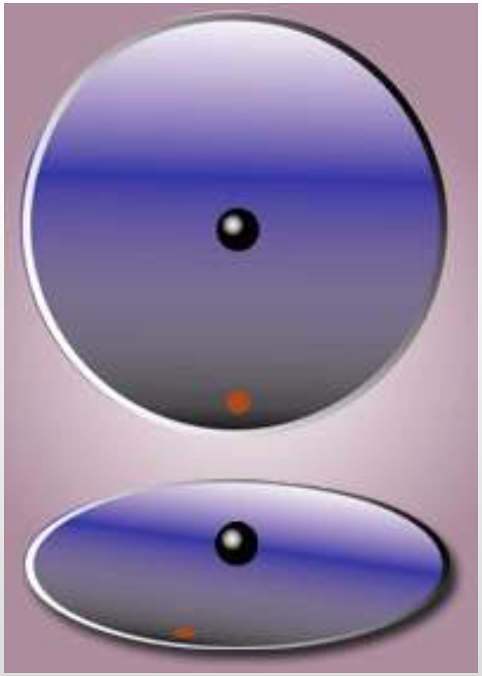
$$A_{slip} = \ddot{b}e^{j\theta_3}$$

$$A_{AB\,slip} = 3327.9e^{j91.4} \quad in/s^2$$

$$A_{AB\,slip} = 3327.9 \quad @ \quad 91.4^o$$

Coriolis Effect

- The **Coriolis** force is an inertial force that acts on objects that are in motion relative to a rotating reference frame.
- In a reference frame with clockwise rotation, the force acts to the left of the motion of the object.
- In a reference frame with counterclockwise rotation, the force acts to the right of the motion of the object



https://en.wikipedia.org/wiki/Coriolis_force

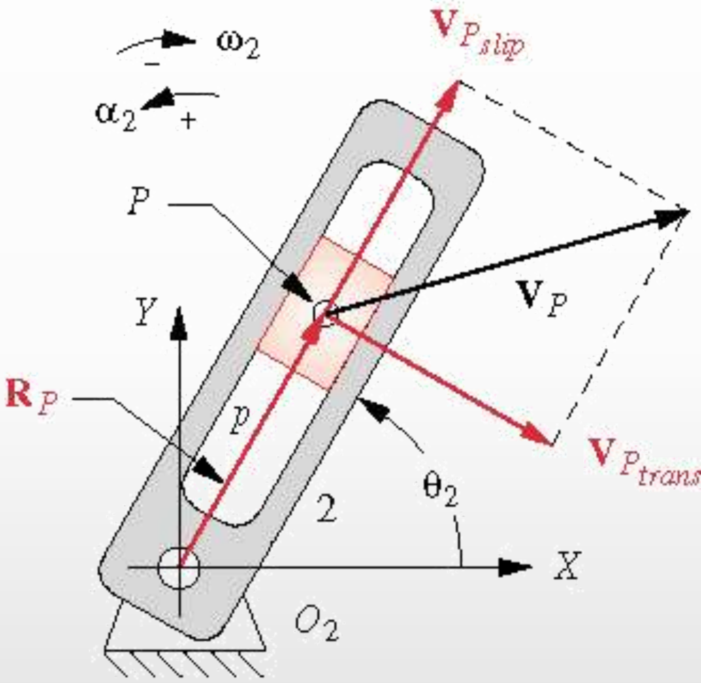
Coriolis Acceleration

Position: $R_P = pe^{j\theta_2}$

Velocity: $V_P = \dfrac{dR_P}{dt} = p\omega_2 je^{j\theta_2} + \dot{p}e^{j\theta_2}$

Transmission velocity: $V_{P_{trans}} = p\omega_2 je^{j\theta_2}$

Slip velocity: $V_{P_{slip}} = \dot{p}e^{j\theta_2}$



$$V_P = V_{P_{trans}} + V_{P_{slip}}$$

Coriolis Acceleration

Acceleration:

$$A_P = \frac{dV_P}{dt} = \frac{d}{dt}\left(p\omega_2 je^{j\theta_2} + \dot{p}e^{j\theta_2}\right)$$

$$\frac{d}{dt}\left(p\omega_2 je^{j\theta_2}\right) = p\alpha_2 je^{j\theta_2} + p\omega_2^2 j^2 e^{j\theta_2} + \dot{p}\omega_2 je^{j\theta_2}$$

$$\frac{d}{dt}\left(\dot{p}e^{j\theta_2}\right) = \dot{p}\omega_2 je^{j\theta_2} + \ddot{p}e^{j\theta_2}$$

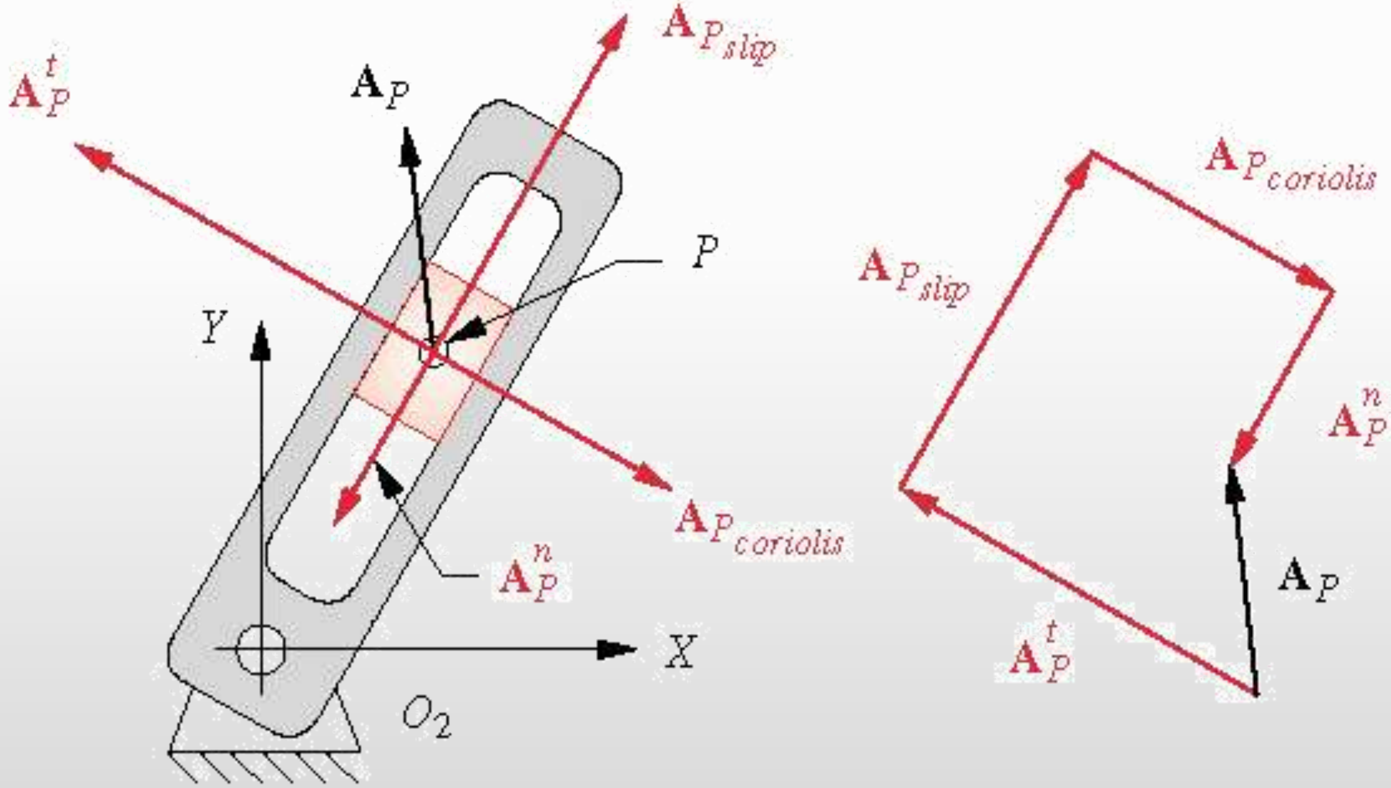
$$A_P = \left(p\alpha_2 je^{j\theta_2} + p\omega_2^2 j^2 e^{j\theta_2} + \dot{p}\omega_2 je^{j\theta_2}\right) + \left(\dot{p}\omega_2 je^{j\theta_2} + \ddot{p}e^{j\theta_2}\right)$$

$$A_P = p\alpha_2 je^{j\theta_2} - p\omega_2^2 e^{j\theta_2} + 2\dot{p}\omega_2 je^{j\theta_2} + \ddot{p}e^{j\theta_2}$$

$$A_P = A_{P_{tangential}} + A_{P_{normal}} + A_{P_{coriolis}} + A_{P_{slip}}$$

Coriolis Acceleration

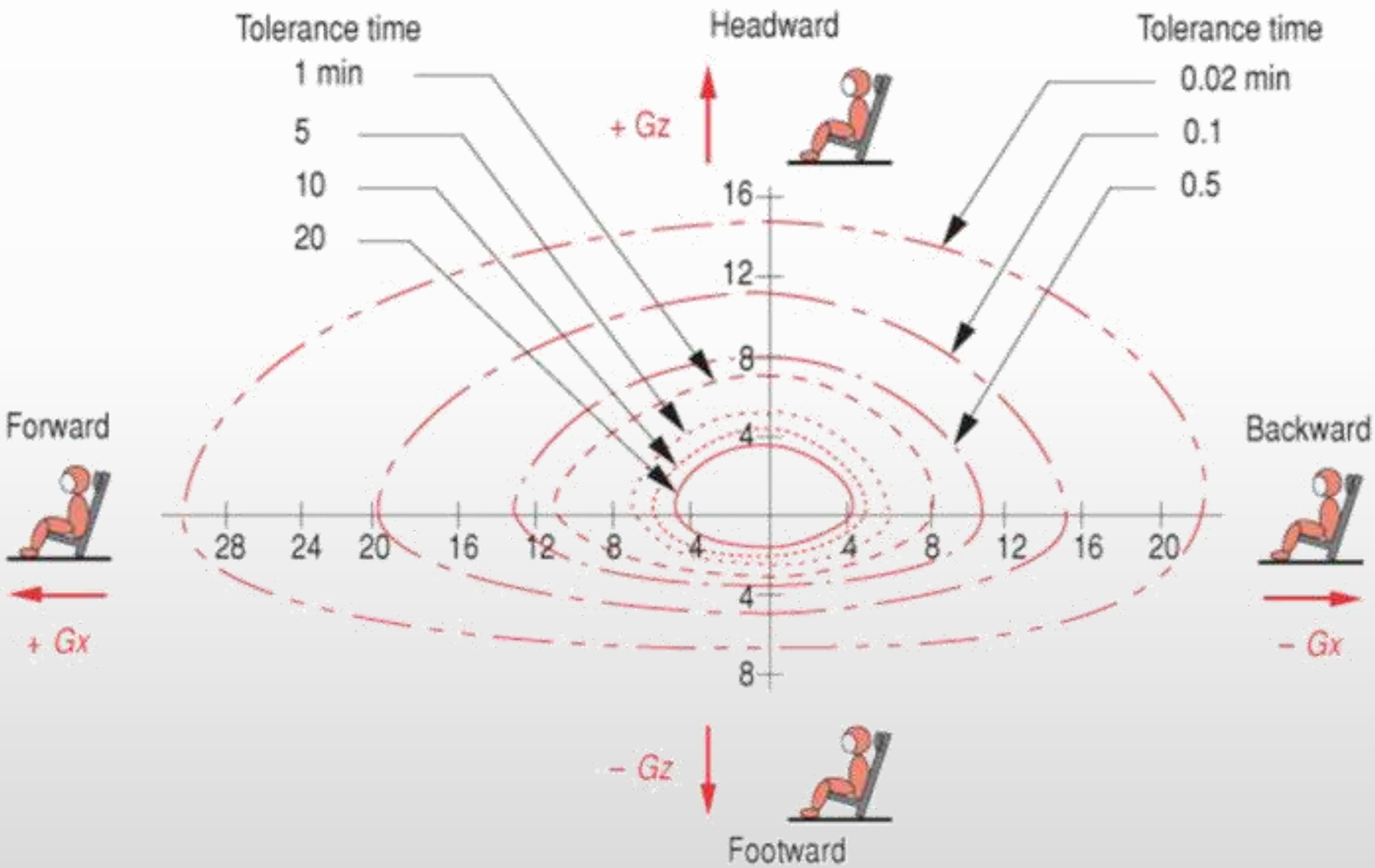
$$A_P = A_{P_{tangential}} + A_{P_{normal}} + A_{P_{coriolis}} + A_{P_{slip}}$$



Humans and acceleration

- The human body senses velocity only by sight whereas the body is very sensitive to acceleration: Canals in human ears are sensitive accelerometers
- Example: when an elevator stops & starts, the change in acceleration produces dynamic forces ($F = ma$)
- Since the human body is not rigid, forces may be very harmful: upward and downward accelerations can flood (red out) or starve (black out) your brain with/from blood.

Human Tolerance of Acceleration



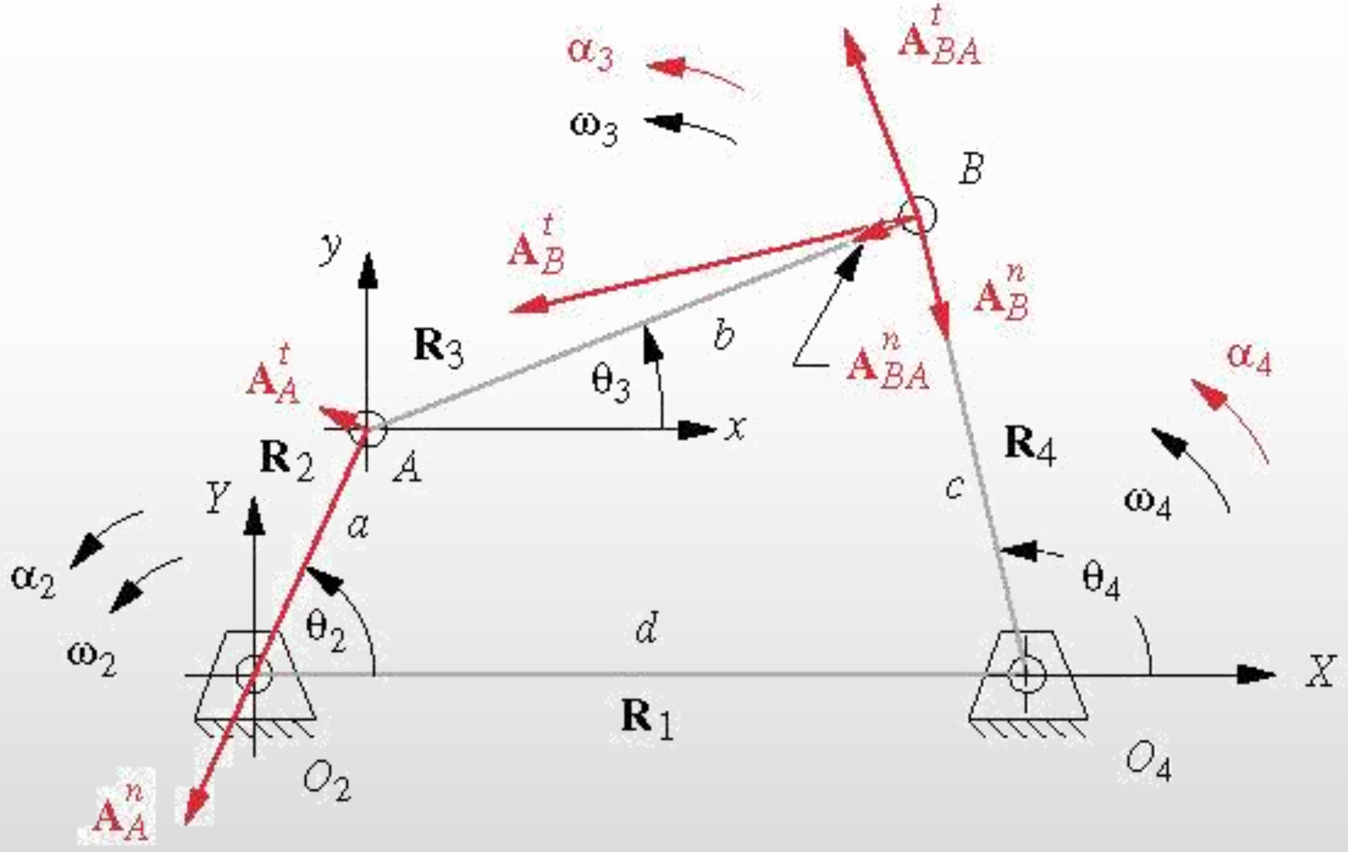
Jerk

- Jerk is defined as the rate of change of acceleration (a.k.a pulse or shock)

- Jerk may be Angular or Linear

$$AngularJerk = \varphi = \frac{d\alpha}{dt} \qquad LinearJerk = J = \frac{dA}{dt}$$

Jerk in 4bar Linkage



Jerk in 4bar Linkage

$$A = a\omega_2^3 \sin\theta_2 \qquad D = b\omega_3^3 \sin\theta_3 \qquad G = 3c\omega_4\alpha_4 \cos\theta_4$$

$$B = 3a\omega_2\alpha_2 \cos\theta_2 \qquad E = 3b\omega_3\alpha_3 \cos\theta_3 \qquad H = c\sin\theta_4$$

$$C = a\varphi_2 \sin\theta_2 \qquad F = c\omega_4^3 \sin\theta_4 \qquad K = b\sin\theta_3$$

$$L = a\omega_2^3 \cos\theta_2 \qquad P = b\omega_3^3 \cos\theta_3 \qquad S = c\omega_4^3 \cos\theta_4$$

$$M = 3a\omega_2\alpha_2 \sin\theta_2 \qquad Q = 3b\omega_3\alpha_3 \sin\theta_3 \qquad T = 3c\omega_4\alpha_4 \sin\theta_4$$

$$N = a\varphi_2 \cos\theta_2 \qquad R = b\cos\theta_3 \qquad U = c\cos\theta_4$$

$$\varphi_3 = \frac{A - B - C + D - E - F + G + H\varphi_4}{K}$$

$$\varphi_4 = \frac{K\left(N - L - M - P - Q + S + T\right) + R\left(A - B - C + D - E + F + G\right)}{KU - HR}$$